

[1]

- 3.32. (a) Consider the system of Figure P3.32. The input signal $x(t) = 5$ is applied at $t = 0$. Find the value of $y(t)$ at a very long time after the input is applied.

(i) $H(s) = \frac{4}{s + 5}$ (ii) $H(s) = \frac{s + 5}{s^2 + 2s + 5}$

- (b) Repeat Part (a) for the input signal $x(t) = e^{-3t}u(t)$.

[2]

- 4.4. A periodic signal $x(t)$ is expressed as an exponential Fourier series:

$$x(t) = \sum_{k=-\infty}^{\infty} C_k e^{jk\omega_0 t}$$

Show that the Fourier series for $\hat{x}(t) = x(t - t_0)$ is given by

$$\hat{x}(t) = \sum_{k=-\infty}^{\infty} \hat{C}_k e^{jk\omega_0 t}$$

in which

$$|\hat{C}_k| = |C_k| \quad \text{and} \quad \angle \hat{C}_k = \angle C_k - k\omega_0 t_0$$

[3]

- 4.5. For a real periodic signal $x(t)$, the *trigonometric form* of its Fourier series is given by

$$x(t) = A_0 + \sum_{k=1}^{\infty} [A_k \cos k\omega_0 t + B_k \sin k\omega_0 t]$$

Express the exponential form Fourier coefficients C_k in terms of A_k and B_k .

[4]

- 4.6. This problem will help illustrate the orthogonality of exponentials. Calculate the following integrals:

(a) $\int_0^{2\pi} \sin^2(t) dt$

(b) $\int_0^{2\pi} \sin^2(2t) dt$

(c) $\int_0^{2\pi} \sin(t) \sin(2t) dt$

- (d) Explain how the results of parts (a), (b), and (c) illustrate the orthogonality of exponentials.

$(\frac{1}{T_0}) \int_{T_0} e^{jk\omega_0 t} \cdot e^{jn\omega_0 t} dt = 0 \text{ for } k \neq n$